Further Evidence on the Turn-of-the-Month Effect

Erhard Reschenhofer
Department of Statistics and Decision Support Systems, University of Vienna, A-1010 Austria

Correspondence: erhard.reschenhofer@univie.ac.at, e_reschenhofer@yahoo.com
Published online: September 12, 2010

Abstract

This paper points out that even distinct patterns in financial time series, which persist over long periods of time, cannot immediately be taken as genuine. In view of the large number of possible patterns, the only way to avoid any data- snooping bias is to use a formal statistical test, which has not been tailored to the specific patterns present in the data. Adopting a universal frequency domain test for the detection of synchronous cycles, we find clear evidence for within-month patterns in daily returns on the S&P 500 index, which corroborates earlier findings obtained simply by comparing different days of the month.

Keywords: Data snooping; Synchronous cycles; Optimal test for periodicities.

1. Introduction

Using daily U.S. returns for the 80-year period of 1926–2005, McConnell and Xu [1] examined the turn-of-the-month effect (returns appear to be higher around the turn of the month). They found that this effect, which had already been documented for the period of 1963-1981 by Ariel [2] and for the period of 1897–1986 by Lakonishok and Smidt [3], persisted in the subsequent two decades. While there is in general a strong link between stock prices and real economic activity (only the direction of the causality is disputed [4]), McConnell and Xu [1] found no evidence that the turn-of-the-month effect could be linked to the payday hypothesis (increased trading volume and increased flow to equity mutual funds after the payment of wages [5]). They established the robustness of their findings by examining sub-samples as well as international data, eliminating extreme observations, and taking conditional heteroscedasticity into account. However, there is one remaining concern, namely the possibility of a data-snooping bias, which needs to be addressed in a formal way. In this paper, I will first point out that any fair comparison of different trading days in a month must take into account that the selection of these days is possibly based on historical information. To avoid any data-snooping error I will then adopt the frequency domain test proposed by Ploberger and Reschenhofer [6] for the detection of synchronous cycles in multiple time series. This approach assumes no prior knowledge of the form of the monthly pattern.

2. Possible data-snooping biases

For my empirical investigation I use daily returns on the S&P 500 index from June 02, 1952 to June 30, 2010. The start of the sample period is dictated by the fact that Saturday trading existed until May 24, 1952. Excluding the six months with less than n=18 trading days (September 2001 etc.) there remain k=691 months for the analysis. For each month *j=1,...,k=691*, we construct a time series $r_1(j), \dots, r_n(j)$ of length *n=18* consisting of the first nine and the last nine returns in the month. Possible surplus trading days in the middle of the month are discarded. Figure 1.a shows the cumulative returns

$$\sum_{i=1}^{j} r_t(i), \; j = 1,\dots,k,$$

for each trading day t=1,...,n. Apparently, the returns are higher for the first three and, to a lesser extent, the last three trading days in a month. The n series of cumulative returns shown in Figure 1.a, which have been obtained from the n original series

$$\begin{gathered} r_1(1), \dots, r_1(k), \\ \vdots \\ r_n(1), \dots, r_n(k), \end{gathered}$$

differ considerably from the n synthetic series of cumulative returns shown in Figure 1.b, which have been obtained from the n randomly permuted series

$$r_{\pi_1(1)}(1), \ldots, r_{\pi_k(1)}(k),$$
$$\vdots$$
$$r_{\pi_1(n)}(1), \ldots, r_{\pi_k(n)}(k).$$

The fact that the returns are permuted only within a month guarantees that the heteroscedasticity in the original series is preserved to some extent. However, the random permutations $\pi_1, \ldots, \pi_k$ safely destroy any monthly pattern. Using different random permutations we obtain additional sets of synthetic series (see Figures 1.c-1.f). The obvious difference in the appearance of the series in Figure 1.a and the synthetic series in Figures 1.b-1.f is an indication of the presence of a monthly pattern in the S&P 500 returns. However, the most extreme synthetic series in Figure 1.c raises concern about statistical significance (note that this series "performs" very well both in the first half and the second half of the sample period). What types of extremes can we expect to see when we look at one hundred sets of synthetic series? Figure 2 compares the best of the 18 series in Figure 1.a (with respect to overall performance) to the respective best of 100 sets of 18 synthetic series. The fact that the best of the original series outperforms almost all of the 100 best synthetic series suggests that the hypothesis of no monthly pattern can be rejected at a reasonable level of significance. This finding can easily be corroborated in a robust way by drastically increasing the number of sets of synthetic series (for 10,000 sets we obtain p= 0.023). Using the most extreme day for the construction of a test is certainly much less critical than choosing a "fixed" day (e.g., the first trading day in a month) based on historical data. However, we must still reckon with the possibility of a data-snooping error because there are numerous other ways to construct a test. For example, we might use the best (or worst) N days, the best N and the worst M days, the best (or worst) successive N days, the best N and the worst M successive days, and so on. Clearly, we must strictly avoid tailoring the test to the specific patterns present in the data. We will therefore next take a more robust approach to safely establish the presence of a monthly pattern.

3. Testing for within-month patterns

A robust approach to test for an arbitrary periodic pattern, which is not prone to any data-snooping bias, is to adopt the frequency domain test proposed by Ploberger and Reschenhofer [6]. This test is optimal in certain situations where the periodic patterns are synchronous. Moreover, it is completely insensitive to peaks in the absolutely continuous part of the spectrum. Let

$$\hat{a}_p(j) = \frac{2}{n}\sum_{t=1}^{n} r_t(j)\cos(\lambda_p t), \; p=1,\ldots,m,$$
$$\hat{b}_p(j) = \frac{2}{n}\sum_{t=1}^{n} r_t(j)\sin(\lambda_p t), \; p=1,\ldots,m,$$

be the least squares estimates of the parameters of the hidden periodicities model

$$r_t(j) = \sum_{p=1}^{m}\left(a_p(j)\cos(\lambda_p t) + b_p(j)\sin(\lambda_p t)\right) + u_t(j),$$

where $\lambda_p = \frac{2\pi p}{n}$ and $m \geq 1$. The inclusion of higher-order harmonics $\lambda_2, \ldots, \lambda_m$ in addition to the fundamental frequency λ_1 provides for possible deviations from sinusoidality. In contrast to the frequency λ_1, which represents a monthly pattern, the frequencies $\lambda_2, \ldots, \lambda_m$ usually describe only non-sinusoidality in the monthly pattern rather than independent cyclical behavior (because they are just multiples of λ_1).

Figure 1: (a) Cumulative return on the S&P 500 index for the first nine and the last nine trading days of a month (fat dark-gray lines: 3rd day: dark, 2nd day: darker, 1st day: darkest; thin dark-gray lines: last day: dark, 2nd last day: darker, 3rd last day: darkest). (b)-(f) Synthetic series obtained by randomly permuting the 18 returns within each month.

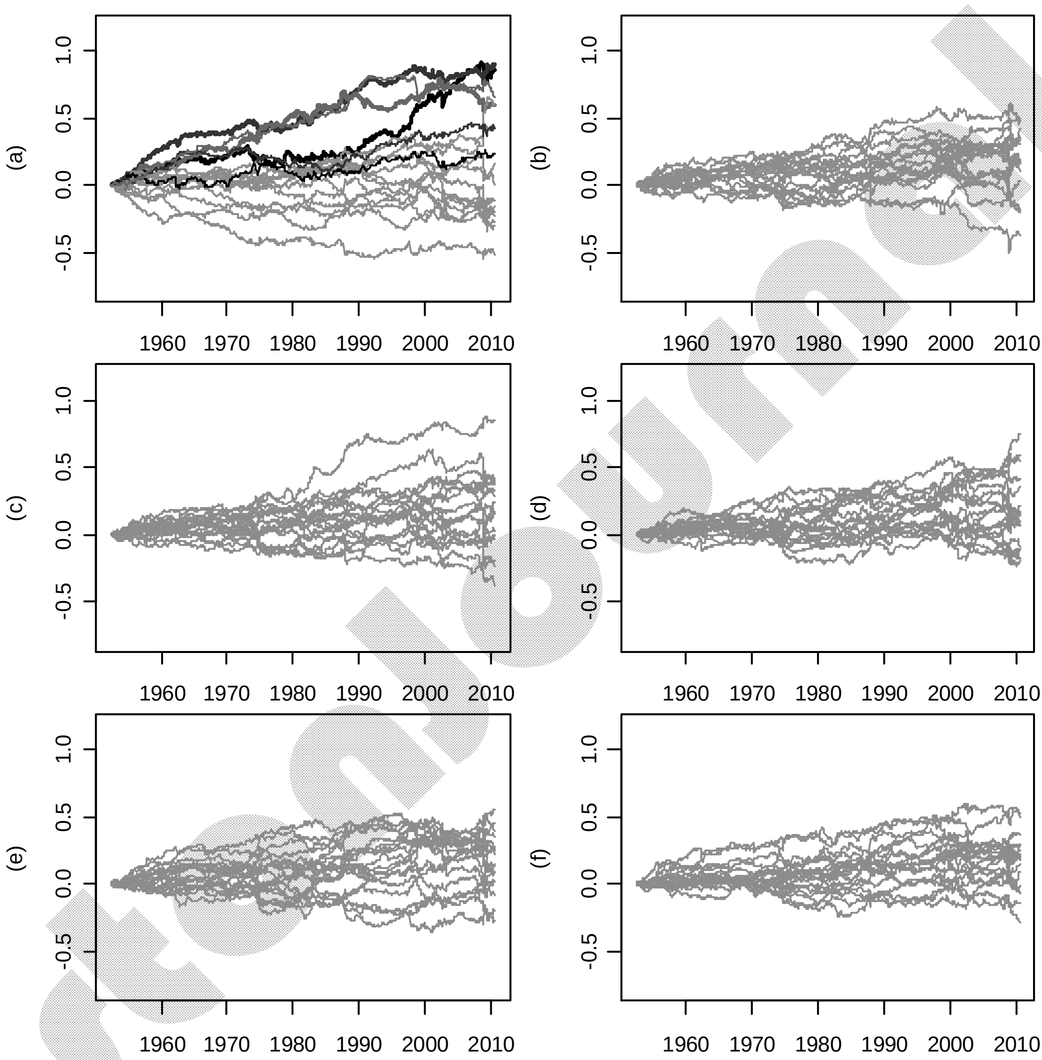
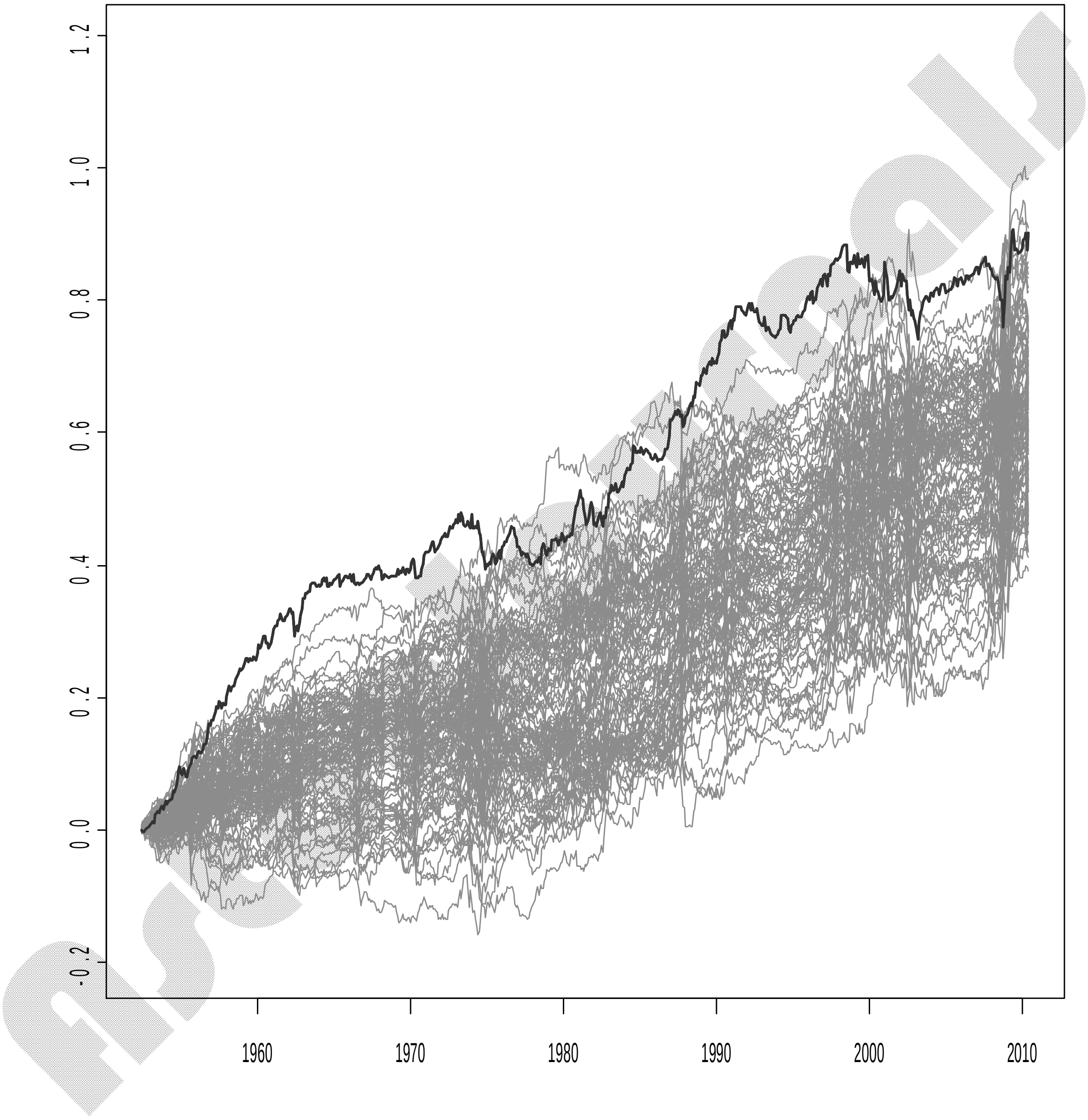


Figure 2: Comparison of the best of the 18 series shown in Figure 1.a (with respect to overall performance) to the respective best of 100 sets of 18 synthetic series.



Under the null hypothesis

$$H_0:\ a_1(j) = ... = a_m(j) = b_1(j) = ... = b_m(j) = 0, \quad j = 1,...,k,$$

the test statistic

$$T = 2\sum_{p=1}^{m} \frac{\left(\sum_{j=1}^{k} \hat{a}_p(j)\right)^2 + \left(\sum_{j=1}^{k} \hat{b}_p(j)\right)^2}{\sum_{j=1}^{k} \hat{a}_p(j)^2 + \sum_{j=1}^{k} \hat{b}_p(j)^2}$$

has approximately a χ^2-distribution with *2m* degrees of freedom if $m<\frac{n}{2}$ [6]. The terms occurring in the numerator will only be large if the k time series contain periodic oscillations with concordant phases. In our application (691 series of 18 daily returns on the S&P 500 index), the test result is significant at the 0.1% level for all values of *m*, i.e., $m=1,...,[\frac{n}{2}]$. Note that if $m=\frac{n}{2}$, $b_m(j)$ vanishes and the degrees of freedom therefore decrease from *2m* to *2m-1*. Note also that our test is quite robust to heteroscedasticity because the phase information is extracted locally from short sub-series of length 18.

4. Conclusion

We have now established the presence of a significant monthly pattern in daily S&P 500 returns in two different ways. In neither case has historical information been used about which days of the month are potentially good or bad. However, the first approach depends critically on the specification of the number of potentially good or bad days. Thus, the significant result obtained later by evaluating the test statistic T for the daily returns on the S&P 500 index is more reliable than the first one. Overall, our results strongly corroborate the existence of the turn-of-the-month effect.

Competing Interests

The author declares that he has no competing interests.

Acknowledgement

The author wishes to thank an anonymous reviewer for helpful comments.

References

[1] McConnell JJ, Xu W, 2008. Equity Returns at the Turn of the Month. Financial Analysts Journal, 64: 49-64.

[2] Ariel RA, 1987. A monthly effect in stock returns. Journal of Financial Economics, 18: 161-174.

[3] Lakonishok J, Smidt S, 1988. Are Seasonal Anomalies Real? A Ninety-Year Perspective. Review of Financial Studies, 1: 403–425.

[4] Adamopoulos A, 2010. Stock Market and Economic Growth: An Empirical Analysis for Germany. Business and Economics Journal, Vol 2010: BEJ-1.

[5] Ogden JP, 1990. Turn-of-Month Evaluations of Liquid Profits and Stock Returns: A Common Explanation for the Monthly and January Effects. Journal of Finance, 45: 1259–1272.

[6] Ploberger W, Reschenhofer E, 2010. Testing for Cycles in Multiple Time Series. Journal of Time Series Analysis, DOI: 10.1111/j.1467-9892.2010.00675.x.